NOTATION

α, β, γ, δ, critical indices; A, A', A_1, A_2, B, Γ, Γ', D, critical amplitudes; a, k, b, coefficients of the linear model of the parametric equation of state; r, θ, parameteric variables determining the distance to the critical point and the path by which it is approached; μ, chemical potential; ρ, density; T, temperature; p, pressure; $Z_{cr} = p_{cr}v_{cr}/RT_{cr}$, compressibility coefficient at the critical point; R, universal gas constant.

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MEASUREMENT OF THE THERMAL DIFFUSIVITY IN CONDITIONS OF SUBSONIC HEATING. CALCULATION OF DYNAMIC CORRECTION

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Corrections associated with the rate of change in mean temperature are calculated for the dynamic method of plane temperature waves. It is shown that the correction is only significant close to the points of phase transition and at heating rates above 1000 K/sec.

The dynamic method of plane temperature waves was proposed in [1] for the investigation of the thermophysical characteristics of materials. A sample in the form of a thin plane-parallel plate is heated at a rate of up to 1000 K/sec with a modulation period of the temperature wave of no more than 10 msec, which allows the thermal diffusivity to be measured over a broad temperature range in a time of less than 1 sec. The creation of this method permits a reduction by two or three orders of magnitude in the time to measure the temperature dependence of the thermal diffusivity and allows informtion to be obtained in the temperature range where the sample cannot retain its form and state for a long time, i.e., close to phase and structural transformations. However, the possibility of using the information obtained for determining the temperature dependence of the thermophysical characteristics in this temperature range requires separate theoretical investigation. The point is that all the traditional nonsteady methods are based on solving linear or linearized heat-conduction equation and therefore the working region of the temperature intervals is always limited. Expansion of the region of application of nonsteady methods requires the solution of complex nonlinear heat-conduction equations, taking account of all the factors responsible for this nonlinearity. Analytical methods of solving problems of this type have not yet been adequately developed. At the same time, computer solution of such problems by numerical methods does not present any difficulties.

All the results of the present work are obtained using a "machine" experiment: essentially, the phhsical process used to measure the thermophysical coefficients is replaced by

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a mathematical model and the quantities bearing information on the process are determined on a computer. In addition, this scheme allows the error in themeasurement method associated with its relization on a specific apparatus (edge effects, nonisothermal conditions over the area of the sample, etc.) to be taken into account

The basic idea of the dynamic method of plane temperature waves for measuring the thermal diffusivity is as follows. On one side of an infinite plane-parallel plate, there acts a time-varying flux

$$q(\tau) = q_0 + V\tau + q_m \cos(2\pi\nu\tau),$$

exciting a temperature wave, which propagates to the opposite boundary. At this boundary, the temperature variation and phase shift between the oscillatiosn of temperature and heat flux is recorded. Both these quantities bear information on the temperature dependence of the thermal diffusivity. At the boundaries of the plate, radiant heat transfer occurs. Initially, the heat fluxes from the boundaries are compensated by the heat flux q_0. There are no convective heat fluxes.

For highly intensive nonsteady heat-transfer processes, it must be taken into account that the heat propagates at a rate which, although very large, is still finite (the rate of heat propagation is eual to the sound velocity according to literature data). Heat transfer is described here by the generalized Fourier law [2]

$$q = -\lambda \frac{\partial T}{\partial x} - \tau_r \frac{\partial q}{\partial \tau}. \quad (1)$$

The relaxation time τ_r appearing in this expression is related to the rate of heat propagation v as $\tau_r = a/v^2$; for metals, for example, iron, it is around 10^{-11} sec. The characteristic time of variation in heat-flux density in this case is the modulation period of temperature oscillations and, since $\tau_{ch} \gg \tau_r$, the second term on the right-hand side of Eq. (1) may be neglected.

Under the given assumptions, the process is modeled by the following boundary problem for the equations of nonlinear heat conduction [2]

$$c(T)\gamma(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right], \quad (2)$$

$$-\lambda(T)\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_0 + V\tau + q_m\cos(2\pi\nu\tau) - \varepsilon\sigma(T^4 - T_{0e}^4), \quad (3)$$

$$-\lambda(T)\left.\frac{\partial T}{\partial x}\right|_{x=\delta} = \varepsilon\sigma(T^4 - T_{0e}^4), \quad (4)$$

$$T(x, 0) = T_{10}(x). \quad (5)$$

The solution is constructed using an implicit finite-difference approximation scheme [3] for the system in Eqs. (2)-(5)

$$\frac{c(T_n)\gamma(T_n)}{\Delta\tau}\Delta x_n(T_n - T_n^{-\Delta\tau}) = \frac{(T_{n+1} - T_n)}{\left(\frac{\Delta x_{n+1}}{2\lambda_{n+1}} + \frac{\Delta x_n}{2\lambda_n}\right)} + \frac{(T_n - T_{n-1})}{\left(\frac{\Delta x_n}{2\lambda_n} + \frac{\Delta x_{n-1}}{2\lambda_{n-1}}\right)}, \quad (6)$$

$$\frac{\lambda(T_0)}{\Delta x_0}(T_0 - T_{-1}) = \varepsilon\sigma(T_{-1/2}^{-\Delta\tau})^3 T_{-1/2} - \varepsilon\sigma T_{0e}^4, \quad (7)$$

$$\frac{\lambda(T_N)}{\Delta x_N}(T_N - T_{N-1}) = q_0 + V\tau + q_m\cos(2\pi\nu\tau) - \varepsilon\sigma(T_{N-1/2}^{-\Delta\tau})^3 T_{N-1/2} + \varepsilon\sigma T_{0e}^4, \quad (8)$$

where n = 0, 1, 2, ..., N − 1.

This scheme is absolutely stable, does not introduce limitations on the relation between the calculation-scheme steps, and approximates the initial Eq. (2) with an error of order $O(\Delta x^2)$. The system of algebraic Eqs. (6)-(8) is solved using the fitting method [3]. This computational algorithm is convenient in that its realization results in a small number of arithmetical operations (reduces the computation time) and has weak sensitivity to the computational error.

The temperature wave is isolated by subtracting from the general solution of the problem in Eqs. (2)-(5) the solution of the sample problem with zero amplitude of the heat-flux oscillations. The result is subjected to harmonic analysis.

Thus, knowing the law of heat-flux variation and specifying the dependence of the thermophysical properties on the temperature and plate geometry, the nonsteady temperature fields and angle of phase shift between the oscillations of the heat flux and the phase lag of the temperature wave at any moment of time may be obtained. The error in determining the phase shift depends basically on the error in approximating the system in Eqs. (2)-(4). Decrease in the division step of the coordinate Δx and time $\Delta\tau$ leads, on the one hand, to decrease in the error and, on the other, to increase in the overall time of machine calculation. Therefore, the optimal conditions of calculation are chosen so that the results of analytical calculation of the phase shift of the linear heat-conduction problem and the results of numerical solution of this problme difer by no more than 1%.

Calculations are made for various ranges of parameter variation: heat flux velocity $V = 0\text{-}2\cdot10^{7}$ $W/m^2\cdot sec$; oscillation frequency of heat flux $\nu = 100\text{-}200$ Hz; thermal conductivity $\lambda = 10\text{-}250$ W/m·K when $(1/\lambda)(\partial\lambda/\partial T) = 0\text{-}0.05\ K^{-1}$; the specific heat $c = 0.04\text{-}4$ kJ/kg·K when $(1/c)(\partial c/\partial T) = 0\text{-}0.05\ K^{-1}$; the temperature $T = 1000\text{-}3000$ K; emissivity $\varepsilon = 0\text{-}1$.

The plate thickness is $1.8\cdot10^{-4}$ m. The mean heating rate is in the range 0-5000 K/sec. The maximum values of the heat flux $q(\tau)$ and phase shift φ are no more than $5\cdot10^{6}$ W/m^2 and 2π rad, respectively. The amplitude of heat-flux oscillations is taken to be $5\cdot10^{4}$ W/m^2.

The Stark number, charaterizing the relation between the temperature field in the solid and the conditions of radiant heat transfer at its surface

$$\mathrm{St} = \frac{\varepsilon\sigma T^3\delta}{\lambda} < 2\cdot10^{-2}$$

for the whole range of variation of the parameters appearing there.

The primary aim of the given numerical experiments is to elucidate the functional dependence of the phase shift betrween the heat-flux oscillations and the temperature oscillations at the surface opposite to the heated surface on the dimensional parameter

$$\mathrm{Pd} = \frac{2\pi\nu}{a}\delta^2$$

in the case where the heat conduction and specific heat depend continuously on the temperature, and the thermal diffusivity remains constant

$$a(T+\Delta T) = a(T),\ \Delta T \to 0, \tag{9}$$

while the functions $\lambda(T)$ and $c(T)$ have a continuous first derivative with respect to the temperature. Using the definition of the thermal diffusivity for the given temperature in the form

$$a(T) = \frac{\lambda(T)}{\gamma c(T)},$$

it is found that Eq. (9) is satisfied when

$$\frac{1}{\lambda}\frac{\partial\lambda}{\partial T} = \frac{1}{c}\frac{\partial c}{\partial T}.$$

The dependence $\mathrm{Pd} = f(\varphi, \mathrm{St})$ calculated in this way for different heating rates completely coincides with the analogous dependences obtained for quasi-steady conditions and constnat values of λ, c, a, [4]. This allows the relation

$$a_T(T) = \frac{2\pi\nu\delta^2}{(1{,}414\varphi(T) - 1{,}11)^2}, \tag{10}$$

obtained in [4] to be used in subsequent numerical experiments to determine the temperature dependence of the thermal diffusivity with respect to the phase shift of the temperature wave. The specified function $a(T)$ will play the role of true thermal-diffusivity values here, and the dynamic correction takes the form of the difference between $a(T)$ and $a_T(T)$, i.e., $\tilde{a}(T) = a(T) - a_T(T)$.

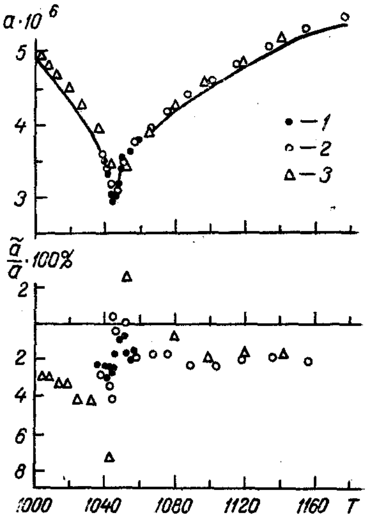

Fig. 1. Dependence of the thermal diffusivity and the relative error on the temperature; the continuous curve correspond to the data of [5] and the points to theoretical values: 1) $V = 4\cdot10^6$ W/m$^2\cdot$sec, $T_0 = 1040$ K; 2) $4\cdot10^7$, 1040; 3) $4\cdot10^7$, 1000; below the T axis, $\tilde{a}/a$ is negative.

It follows from the complete set of calculations above that the dynamic correction is determined by a combination of all the specified parameters and, in addition, by the method of determining the temperature corresponding to the phase shift which appears in Eq. (10). In experimental measurement of the thermal diffusivity in dynamic conditions, the method of plane temperature waves is used to measure the temperature at the surface opposite the heated surface. Therefore, taking the mean temperature of this surface over the period of heat-flux oscillation as the reference temperature, the dynamic correciton may be estimated from the formula

$$\tilde{a}(T) = \frac{1}{2}\,\frac{\partial a}{\partial T}\left[\frac{V_T}{\nu} + \frac{q(\tau)\,\delta}{2\lambda(T)}\right]. \tag{11}$$

The first term in the square brackets is the temperature variation over time in the modulation period and the second is the temperature difference over the plate thickness, which is significant with unilateral heating at a high temperature level, even for thin metallic samples, and may exceed the temperature variation in the modulation period.

For example, for $\nu = 160$ Hz, $a_0 = 3.52\cdot10^{-6}$ m^2/sec, $\lambda_0 = 15$ W/m·K, $T_0 = 1773$ K, $V = 4\cdot10^6$ W/m$^2\cdot$sec, $(1/a_0)(\partial a/\partial T) = 0.01$, $(1/\lambda_0)(\partial\lambda/\partial T) = 0.01$, and mean heating rate ~1600 K/sec, the temperature difference over the plate thickness is ~10 K. Numerical experiment and calculation by Eq. (11) give $\tilde{a}(T)/a(T)$ of ~6 and ~10%, respectively.

The temperature dependence of the thermal diffusivity of iron close to the Curie point is shown in Fig. 1. The basic values of this coefficient obtained in quasi-steady conditions are taken from [5]. The results of numerical experiment are given for two heat-flux velocities and two initial values of the plate temperature. It is evident that, in the vicinity of the Curie point, for mean heating rates of ~200 and ~1600 K/sec, the error in determining the thermal diffusivity is ~3 and ~7%, respectively. On sections of smooth variation, it is no more than ~3.5%. Note, however, that this error is associated only with the dynamics of the process is an additional error to the result obtained in quasisteady conditions.

Thus, it is found that the method of plane temperature waves in dynamic conditions allows the thermal diffusivity of the materials to be measured close to points of phase transition and structural transformation. The methodological error in the given case depends on the heating temperature and the temperature gradient over the sample thickness, the maximum value of which may be estimated from Eq. (11). In the temperature intervals where $(1/a)(\partial a/\partial T) \ll 0.01$, the methodological error is practically independent of the heating rate and is no more than 2.5% over the whole of the specified range of temperature variation.

NOTATION

q, heat-flux density, W/m^2; V, rate of increase in heat-flux density, $W/m^2 \cdot sec$; V_T, heating rate, K/sec; q_m, amplitude of oscillations in heat-flux density, W/m^2; ν, modulation frequanty, Hz; τ, time, sec; T, temperature, K; T_{0e}, temperature of medium, K; T_{10}, initial temperature distribution, K; q_0, initial heat-flux density, W/m^2; x, coordinate, m; c, specific heat, J/kg·K; γ, density, kg/m^3; λ, thermal conductivity, W/m·K; ε, integral emissivity; σ, Stefan—Boltzmann constant, $W/m^2 \cdot K^4$, n, step number; δ, plate thickness, m; St, Stark number, a, thermal diffusivity; m^2/sec; $\omega = 2\pi\nu$, cyclic frequency, rad/sec; Pd, Predvoditelev number; φ, phase shift, rad.

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DYNAMICS OF THE DRAWING ZONE OF A LIGHTGUIDE BLANK FOR DIFFERENT DRAWING REGIMES WITH FURNACE AND LASER HEATING

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The results of numerical modeling of the process of drawing a quartz blank into a lightguide with different mehtods of heating are presented. The optimal regions of the space of drawing parameters for obtaining a stable lightguide diameter are determined.

Introduction. In preparing fiber lightguides by the method of drawing from a blank (rod) many physical problems must be solved. One particular problem is to investigate the behavior of the zone of drawing between the blank and the lightguide, the so-called "onion." This question is of interest because the basic characteristics of the lightguide obtained are determined precisely by the zone of formation. In particular, the stability of the diameter along the lightguide depends on the character of the oscillation of the onion during the drawing process. For this reason much attention is devoted in the experimental and theoretical works to the behavior of the onion [1-6].

We performed a series of numerical experiments devoted to this question. The process of drawing a quartz blank into a lightguide [9, 10] was modeled by applying to this problem the methods of numerical simulation fo the motion of a viscous incompressible liquid bounded by a "free surface" [7, 8]. The motion of the quartz glass was regarded as a vertical, axisymmetric, nonstationary motion of a liquid bounded by a "free surface." All experimentally recorded situations were simulated, namely, stable continuous drawing of a blank into a lightguide, break off of the lightguide owing to capillary decomposition accompanying overheating of the drawing zone, and break off owing to underheating (viscous fracture). Thus it was established in [10] that an arbitrary combination of technological parameters is suitble for drawing. On the contrary, in the space of technological parametes of the drawing process there exist "allowed" and "forbidden" regions for the operation.

In this paper we investigate the dynamics of the behavior of the onion within the regions of parameters "allowed" for drawing for two basic methods of drawing — laser and furnace heating.

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